

Numbers

NATURAL NUMBERS

{1, 2, 3, 4, 5, ...}

WHOLE NUMBERS

{0, 1, 2, 3, 4, ...}

INTEGERS

{..., -3, -2, -1, 0, 1, 2, ...}

The Number Line



RATIONAL NUMBERS

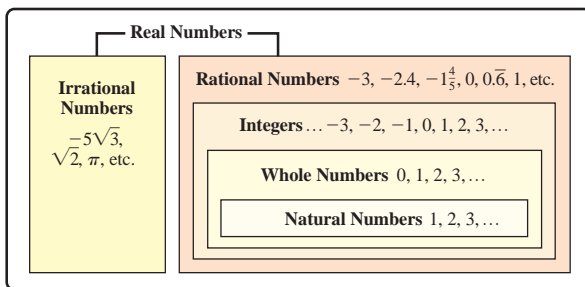
All numbers that can be written in the form a/b , where a and b are integers and $b \neq 0$.

IRRATIONAL NUMBERS

Real numbers that cannot be written as the quotient of two integers but can be represented on the number line.

REAL NUMBERS

Include all numbers that can be represented on the number line, that is, all rational and irrational numbers.



PRIME NUMBERS

A prime number is a number greater than 1 that has only itself and 1 as factors.

Some examples:

2, 3, and 7 are prime numbers.

COMPOSITE NUMBERS

A composite number is a number that is not prime. For example, 8 is a composite number since

$$8 = 2 \cdot 2 \cdot 2 = 2^3.$$

Important Properties

PROPERTIES OF ADDITION

- Identity Property of Zero: $a + 0 = a$
- Inverse Property: $a + (-a) = 0$
- Commutative Property: $a + b = b + a$
- Associative Property: $a + (b + c) = (a + b) + c$

PROPERTIES OF MULTIPLICATION

- Property of Zero: $a \cdot 0 = 0$
- Identity Property of One: $a \cdot 1 = a$, when $a \neq 0$.
- Inverse Property: $a \cdot \frac{1}{a} = 1$, when $a \neq 0$.
- Commutative Property: $a \cdot b = b \cdot a$
- Associative Property: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

PROPERTIES OF DIVISION

- Property of Zero: $\frac{0}{a} = 0$, when $a \neq 0$.
- Property of One: $\frac{a}{a} = 1$, when $a \neq 0$.
- Identity Property of One: $\frac{a}{1} = a \cdot 1$

Absolute Value

The absolute value of a number is always ≥ 0 .

$$\text{If } a > 0, |a| = a.$$

$$\text{If } a < 0, |-a| = a.$$

For example, $|-5| = 5$ and $|5| = 5$. In each case, the answer is positive.

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Key Words and Symbols

The following words and symbols are used for the operations listed.

Addition

Sum, total, increase, plus

$$\text{addend} + \text{addend} = \text{sum}$$

Subtraction

Difference, decrease, minus

$$\text{minuend} - \text{subtrahend} = \text{difference}$$

Multiplication

Product, of, times

$$a \times b, a \cdot b, (a)(b), ab$$

$$\text{factor} \times \text{factor} = \text{product}$$

Division

Quotient, per, divided by

$$a \div b = \frac{a}{b} = a/b = b\overline{)a}$$

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

Order of Operations

1st: *Parentheses*

Simplify any expressions inside parentheses.

2nd: *Exponents*

Work out any exponents.

3rd: *Multiplication and Division*

Solve all multiplication and division, working from left to right.

4th: *Addition and Subtraction*

These are done last, from left to right.

For example,

$$\begin{aligned} 15 - 2 \cdot 3 + (30 - 3) \div 3^2 \\ &= 15 - 2 \cdot 3 + 27 \div 9 \\ &= 15 - 6 + 3 \\ &= 12. \end{aligned}$$

Integers

ADDING AND SUBTRACTING WITH NEGATIVES

$$-a - b = (-a) + (-b)$$

$$-a + b = b - a$$

$$a - (-b) = a + b$$

Some examples:

$$\begin{aligned} -3 - 17 &= (-3) + (-17) = -20 \\ -19 + 4 &= 4 - 19 = -15 \end{aligned}$$

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Integers (continued)

MULTIPLYING AND DIVIDING WITH NEGATIVES

$$-a \cdot b = -ab$$

$$-a \cdot -b = ab$$

$$\frac{-a}{-b} = \frac{a}{b}$$

$$-a \div b = -\frac{a}{b}$$

Some examples:

$$-3 \cdot 5 = -15$$

$$(-7)(-6) = 42$$

$$(-24)/(-8) = 3$$

$$2\overline{) -18} \quad \text{or} \quad \frac{-36}{2} = -18$$

Fractions

LEAST COMMON MULTIPLE

The LCM of a set of numbers is the smallest number that is a multiple of all the given numbers.

For example, the LCM of 5 and 6 is 30, since 5 and 6 have no factors in common.

GREATEST COMMON FACTOR

The GCF of a set of numbers is the largest number that can be evenly divided into each of the given numbers.

For example, the GCF of 24 and 27 is 3, since both 24 and 27 are divisible by 3, but they are not both divisible by any numbers larger than 3.

FRACTIONS

Fractions are another way to express division. The top number of a fraction is called the **numerator**, and the bottom number is called the **denominator**.

ADDING AND SUBTRACTING FRACTIONS

Fractions must have the same denominator before they can be added or subtracted.

$$\frac{a}{d} + \frac{b}{d} = \frac{a + b}{d}, \text{ with } d \neq 0.$$

$$\frac{a}{d} - \frac{b}{d} = \frac{a - b}{d}, \text{ with } d \neq 0.$$

If the fractions have different denominators, rewrite them as equivalent fractions with a common denominator. Then add or subtract the numerators, keeping the denominators the same. For example,

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}.$$

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Fractions (continued)

Equivalent fractions are found by multiplying the numerator and denominator of the fraction by the same number. In the previous example,

$$\frac{2}{3} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{8}{12} \quad \text{and} \quad \frac{1}{4} = \frac{1 \cdot 3}{4 \cdot 3} = \frac{3}{12}.$$

MULTIPLYING AND DIVIDING FRACTIONS

When multiplying and dividing fractions, a common denominator is not needed. To multiply, take the product of the numerators and the product of the denominators:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{ac}{bd}$$

To divide fractions, invert the second fraction and then multiply the numerators and denominators:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Some examples:

$$\frac{3}{5} \cdot \frac{2}{7} = \frac{6}{35}$$

$$\frac{5}{12} \div \frac{1}{2} = \frac{5}{12} \cdot \frac{2}{1} = \frac{10}{12} = \frac{5}{6}$$

REDUCING FRACTIONS

To **reduce** a fraction, divide both the numerator and denominator by common factors. In the last example,

$$\frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}.$$

MIXED NUMBERS

A mixed number has two parts: a whole number part and a fractional part. An example of a mixed number is $5\frac{3}{8}$. This really represents

$$5 + \frac{3}{8},$$

which can be written as

$$\frac{40}{8} + \frac{3}{8} = \frac{43}{8}.$$

Similarly, an improper fraction can be written as a mixed number. For example,

$$\frac{20}{3} \text{ can be written as } 6\frac{2}{3},$$

since 20 divided by 3 equals 6 with a remainder of 2.

Rates, Ratios, Proportions, and Percents

RATES AND RATIOS

A **rate** is a comparison of two quantities with *different* units. For example, a car that travels 110 miles in 2 hours is moving at a rate of 110 miles/2 hours or 55 mph.

A **ratio** is a comparison of two quantities with the *same* units. For example, a class with 23 students has a student–teacher ratio of 23:1 or $\frac{23}{1}$.

PROPORTIONS

A proportion is a statement in which two ratios or rates are equal.

An example of a proportion is the following statement:
30 dollars is to 5 hours as 60 dollars is to 10 hours.

This is written

$$\frac{\$30}{5 \text{ hr}} = \frac{\$60}{10 \text{ hr}}.$$

A typical proportion problem will have one unknown quantity, such as

$$\frac{1 \text{ mile}}{20 \text{ min}} = \frac{x \text{ miles}}{60 \text{ min}}.$$

We can solve this equation by cross multiplying as shown:

$$20x = 60 \cdot 1$$

$$x = \frac{60}{20} = 3.$$

So, it takes 60 minutes to walk 3 miles.

PERCENTS

A percent is the number of parts out of 100. To write a percent as a fraction, divide by 100 and drop the percent sign.

For example,

$$57\% = \frac{57}{100}.$$

To write a fraction as a percent, first check to see if the denominator is 100. If it is not, write the fraction as an equivalent fraction with 100 in the denominator. Then the numerator becomes the percent. For example,

$$\frac{4}{5} = \frac{80}{100} = 80\%.$$

To find a percent of a quantity, multiply the percent by the quantity.

For example, 30% of 5 is

$$\frac{30}{100} \cdot 5 = \frac{150}{100} = \frac{3}{2}.$$

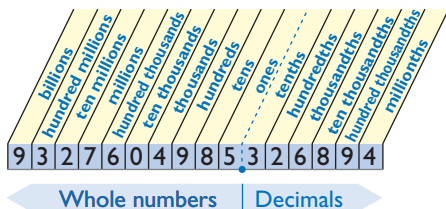
Basic Math Review

Decimal Numbers

The numbers after the decimal point represent fractions with denominators that are powers of 10. The decimal point separates the whole number part from the fractional part.

For example, 0.9 represents $\frac{9}{10}$.

Place Value Chart



ADDING AND SUBTRACTING DECIMAL NUMBERS

To add or subtract decimal numbers, line up the numbers so that the decimal points are aligned. Then add or subtract as usual, keeping the decimal point in the same place.

For example, $23 - 0.37 =$

$$\begin{array}{r} 23.00 \\ - 0.37 \\ \hline 22.63 \end{array}$$

MULTIPLYING AND DIVIDING DECIMAL NUMBERS

To multiply decimal numbers, multiply them as though they were whole numbers. The number of decimal places in the product is the sum of the number of decimal places in the factors. For example, 3.72×4.5 is

$$\begin{array}{r} 3.72 \\ \times 4.5 \\ \hline 16.740 \end{array}$$

2 decimal places
1 decimal place
3 decimal places

To divide decimal numbers, first make sure the divisor is a whole number. If it is not, move the decimal place to the right (multiply by 10, 100, and so on) to make it a whole number. Then move the decimal point the same number of places in the dividend.

For example,

$$\begin{array}{r} 0.42 \div 1.2 = 4.2 \div 12 \\ 0.35 \\ = 12 \overline{)4.20} \end{array}$$

The decimal point in the answer is placed directly above the new decimal point in the dividend.

Percents to Decimals and Decimals to Percents

To change a number from a percent to a decimal, divide by 100 and drop the percent sign:

$$58\% = 58/100 = 0.58.$$

To change a number from a decimal to a percent, multiply by 100 and add the percent sign:

$$0.73 = .73 \times 100 = 73\%.$$

Simple Interest

Given the principal (amount of money to be borrowed or invested), interest rate, and length of time, the amount of interest can be found using the formula

$$I = p \cdot r \cdot t$$

where $I =$ interest (dollar amount)
 $p =$ principal
 $r =$ percentage rate of interest
 $t =$ time period.

For example, find the amount of simple interest on a \$3800 loan at an annual rate of 5.5% for 5 years:

$$\begin{aligned} p &= \$3800 \\ r &= 5.5\% = 0.055 \\ t &= 5 \text{ years} \\ I &= (3800)(0.055)(5) = 1045. \end{aligned}$$

The amount of interest is \$1045.

Scientific Notation

Scientific notation is a convenient way to express very large or very small numbers. A number in this form is written as $a \times 10^n$, where $1 \leq |a| < 10$ and n is an integer. For example, 3.62×10^5 and -1.2×10^{-4} are expressed in scientific notation.

To change a number from scientific notation to a number without exponents, look at the power of ten. If that number is positive, move the decimal point to the right. If it is negative, move the decimal point to the left. The number tells you how many places to move the decimal point.

For example,

$$3.97 \times 10^3 = 3970.$$

To change a number to scientific notation, move the decimal point so it is to the right of the first nonzero digit. If the decimal point is moved n places to the left and this makes the number smaller, n is positive; otherwise, n is negative. If the decimal point is not moved, n is 0.

For example, $0.0000216 = 2.16 \times 10^{-5}$.

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Scientific Notation (continued)

MULTIPLYING AND DIVIDING IN SCIENTIFIC NOTATION

To multiply or divide numbers in scientific notation, we can change the order and grouping, so that we multiply or divide first the decimal parts and then the powers of 10. *For example,*

$$\begin{aligned} & (3.7 \times 10^{-3}) \cdot (2.5 \times 10^8) \\ &= (3.7 \times 2.5) \cdot (10^{-3} \times 10^8) \\ &= 9.25 \times 10^5. \end{aligned}$$

Statistics

There are several ways to study a list of data.

Mean, or average, is the sum of all the data values divided by the number of values.

Median is the number that separates the list of data into two equal parts. To find the median, list the data in order from smallest to largest. If the number of data is odd, the median is the middle number. If the number of data is even, the median is the average of the two middle numbers.

Mode is the number in the list that occurs the most frequently. There can be more than one mode.

For example, consider the following list of test scores:

$$\{87, 56, 69, 87, 93, 82\}$$

To find the mean, first add:

$$87 + 56 + 69 + 87 + 93 + 82 = 474.$$

Then divide by 6:

$$\frac{474}{6} = 79.$$

The mean score is 79.

To find the median, first list the data in order:

$$56, 69, 82, 87, 87, 93.$$

Since there is an even number of data, we take the average of 82 and 87:

$$\frac{82 + 87}{2} = \frac{169}{2} = 84.5.$$

The median score is 84.5.

The mode is 87, since this number appears twice and each of the other numbers appears only once.

Distance Formula

Given the rate at which you are traveling and the length of time you will be traveling, the distance can be found by using the formula

$$d = r \cdot t$$

where d = distance
 r = rate
 t = time

Measurements

U.S. Measurement Units

in. = inch	oz = ounce
ft = foot	c = cup
min = minute	mi = mile
sec = second	hr = hour
gal = gallon	lb = pound
yd = yard	qt = quart
pt = pint	T = ton

Metric Units

mm = millimeter
cm = centimeter
km = kilometer
m = meter
mL = milliliter
cL = centiliter
L = liter
kL = kiloliter
mg = milligram
cg = centigram
g = gram
kg = kilogram

U.S. AND METRIC CONVERSIONS

U.S.

12 in. = 1 ft	3 ft = 1 yd
1760 yd = 1 mi	5280 ft = 1 mi
2 c = 1 pt	1 c = 8 oz
4 qt = 1 gal	2 pt = 1 qt
2000 lb = 1 T	16 oz = 1 lb

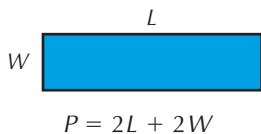
Metric

1000 mm = 1 m	100 cm = 1 m
1000 m = 1 km	100 cL = 1 L
1000 mL = 1 L	100 cg = 1 g
1000 mg = 1 g	1000 g = 1 kg
0.001 m = 1 mm	0.01 m = 1 cm
0.001 g = 1 mg	0.01 g = 1 cg
0.001 L = 1 mL	0.01 L = 1 cL

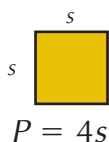
Geometry

The **perimeter** of a geometric figure is the distance around it or the sum of the lengths of its sides.

The perimeter of a rectangle is 2 times the length plus 2 times the width:



The perimeter of a square is 4 times the length of a side:



Area is always expressed in square units, since it is two-dimensional.

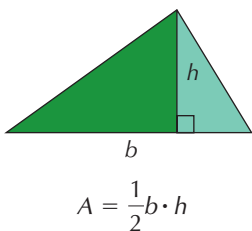
The formula for area of a rectangle is

$$A = L \cdot W.$$

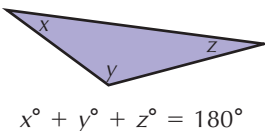
The formula for area of a square is

$$A = s \cdot s \quad \text{or} \quad A = s^2.$$

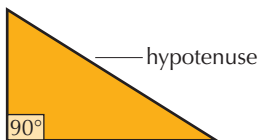
The area of a triangle is one-half the product of the height and base:



The sum of all three angles in any triangle always equals 180 degrees.



A **right triangle** is a triangle with a 90° (right) angle. The **hypotenuse** of a right triangle is the side opposite the right angle.



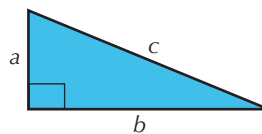
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Geometry (continued)

PYTHAGOREAN THEOREM

In any right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then

$$a^2 + b^2 = c^2.$$

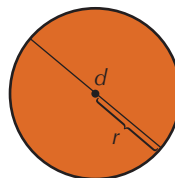


CIRCLES

Area: $A = \pi \cdot r^2$

Circumference: $C = \pi \cdot d = 2 \cdot \pi \cdot r$

where d is the diameter, r is the radius, or half the diameter, and π is approximately 3.14 or $\frac{22}{7}$.



A circle has an angle of 360 degrees.

A straight line has an angle of 180 degrees.

Algebraic Terms

Variable: A variable is a letter that represents a number because the number is unknown or because it can change. For example, the number of days until your vacation changes every day, so it could be represented by a variable, x .

Constant: A constant is a term that does not change. For example, the number of days in the week, 7, does not change, so it is a constant.

Expression: An algebraic expression consists of constants, variables, numerals and at least one operation. For example, $x + 7$ is an expression.

Equation: An equation is basically a mathematical sentence indicating that two expressions are equal. For example, $x + 7 = 18$ is an equation.

Solution: A number that makes an equation true is a solution to that equation. For example, in using the above equation, $x + 7 = 18$, we know that the statement is true if $x = 11$.