Algebra Review

Numbers

FRACTIONS

Addition and Subtraction

i. To add or subtract fractions with the same denominator, add or subtract the numerators and keep the same denominator.

ii. To add or subtract fractions with different denominators, find the LCD and write each fraction with this LCD. Then follow the procedure in step i.

Multiplication

Multiply numerators and multiply denominators.

Division

Multiply the first fraction by the reciprocal of the second fraction.

ORDER OF OPERATIONS

Simplify within parentheses, brackets, or absolute value bars or above and below fraction bars first, in the following order.

i. Apply all exponents.

ii. Perform any multiplications or divisions from left to right.

iii. Perform any additions or subtractions from left to right.

VARIABLES, EXPRESSIONS, AND EQUATIONS

An expression containing a variable is evaluated by substituting a given number for the variable.

Values for a variable that make an equation true are solutions of the equation.

REAL NUMBERS AND THE NUMBER LINE

a is less than b if a is to the left of b on the number line.

The additive inverse of x is $-x$.

The absolute value of x, denoted $|x|$, is the distance (a positive number) between x and 0 on the number line.

OPERATIONS ON REAL NUMBERS

Adding Real Numbers

To add two numbers with the same sign, add their absolute values. The sum has the same sign as each of the numbers being added.

To add two numbers with different signs, subtract their absolute values. The sum has the sign of the number with the larger absolute value.

Definition of Subtraction

$$x - y = x + (-y)$$

Subtracting Real Numbers

i. Change the subtraction symbol to the addition symbol.

ii. Change the sign of the number being subtracted.

iii. Add using the rules for adding real numbers.

Multiplying Real Numbers

i. Multiply the absolute value of the two numbers.

ii. If the two numbers have the same sign, the product is *positve*. If the two numbers have different signs, the product is *negative*.

Definition of Division: $\frac{x}{y} = x \cdot \frac{1}{y}, \; y \neq 0$

Division by 0 is undefined.

Dividing Real Numbers

i. Divide the absolute value of the numbers.

ii. If the signs are the same, the answer is *positive*. If the signs are different, the answer is *negative*.

PROPERTIES OF REAL NUMBERS

Commutative Properties

$$a + b = b + a$$
$$ab = ba$$

Associative Properties

$$(a + b) + c = a + (b + c)$$
$$(ab)c = a(bc)$$

Distributive Properties

$$a(b + c) = ab + ac$$
$$(b + c)a = ba + ca$$

Identity Properties

$$a + 0 = a \qquad 0 + a = a$$
$$a \cdot 1 = a \qquad 1 \cdot a = a$$

Inverse Properties

$$a + (-a) = 0 \qquad (-a) + a = 0$$
$$a \cdot \frac{1}{a} = 1 \qquad \frac{1}{a} \cdot a = 1 \qquad (a \neq 0)$$

Simplifying Algebraic Expressions

When adding or subtracting algebraic expressions, *only like terms* can be combined.

Linear Equations

Properties

i. **Addition:** The same quantity may be added to (or subtracted from) each side of an equality without changing the solution.

ii. **Multiplication:** Each side of an equality may be multiplied (or divided) by the same nonzero number without changing the solution.

Solving Linear Equalities

i. Simplify each side separately.

ii. Isolate the variable term on one side.

iii. Isolate the variable.

APPLICATIONS

i. Assign a variable to the unknown quantity in the problem.

ii. Write an equation involving the unknown.

iii. Solve the equation.

FORMULAS

i. To find the value of one of the variables in a formula, given values for the others, substitute the known values into the formula.

ii. To solve a formula for one of the variables, isolate that variable by treating the other variables as constants (numbers) and using the steps for solving equations.

Exponents

For any integers m and n, the following rules hold:

Product Rule

$$a^m \cdot a^n = a^{m+n}$$

Power Rules

i. $(a^m)^n = a^{mn}$

ii. $(ab)^m = a^m b^m$

iii. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \; b \neq 0$

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Exponents (continued)

Quotient Rules

If $a \neq 0$,

i. Zero exponent: $a^0 = 1$

ii. Negative exponents: $a^{-n} = \frac{1}{a^n}$

iii. Quotient rule: $\frac{a^m}{a^n} = a^{m-n}$

iv. Negative to positive:

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m},\ a \neq 0,\ b \neq 0$$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m,\ a \neq 0,\ b \neq 0$$

Scientific Notation

A number written in scientific notation is in the form $a \times 10^n$, where a has one digit in front of the decimal point and that digit is nonzero. To write a number in scientific notation, move the decimal point to follow the first nonzero digit. If the decimal point has been moved n places to the left, the exponent on 10 is n. If the decimal point has been moved n places to the right, the exponent on 10 is $-n$.

Polynomials

A **polynomial** is an algebraic expression made up of a term or a finite sum of terms with real or complex coefficients and whole number exponents.

The **degree of a term** is the sum of the exponents on the variables. The **degree of a polynomial** is the highest degree amongst all of its terms.

A **monomial** is a polynomial with only *one* term.

A **binomial** is a polynomial with exactly *two* terms.

A **trinomial** is a polynomial with exactly *three* terms.

OPERATIONS ON POLYNOMIALS

Adding Polynomials

Add like terms.

Subtracting Polynomials

Change the sign of the terms in the second polynomial and add to the first polynomial.

Multiplying Polynomials

i. Multiply each term of the first polynomial by each term of the second polynomial.

ii. Collect like terms.

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Polynomials (continued)

FOIL Expansion for Multiplying Two Binomials

i. Multiply the first terms.

ii. Multiply the outer terms.

iii. Multiply the inner terms.

iv. Multiply the last terms.

v. Collect like terms.

SPECIAL PRODUCTS

Square of a Binomial

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

Product of the Sum and Difference of Two Terms

$$(x + y)(x - y) = x^2 - y^2$$

Dividing a Polynomial by a Monomial

Divide each term of the polynomial by the monomial:

$$\frac{p + q}{r} = \frac{p}{r} + \frac{q}{r}$$

Dividing a Polynomial by a Polynomial

Use long division or synthetic division.

Graphing Simple Polynomials

i. Determine several points (ordered pairs) satisfying the polynomial equation.

ii. Plot the points.

iii. Connect the points with a smooth curve.

Factoring

Finding the Greatest Common Factor (GCF)

i. Include the largest numerical factor of each term.

ii. Include each variable that is a factor of every term raised to the smallest exponent that appears in a term.

Factoring by Grouping

i. Group the terms.

ii. Factor out the greatest common factor in each group.

iii. Factor a common binomial factor from the result of step ii.

iv. Try various groupings, if necessary.

Factoring Trinomials, Leading Term = x^2

To factor $x^2 + bx + c,\ a \neq 1$:

i. Find m and n such that $mn = c$ and $m + n = b$.

ii. Then $x^2 + bx + c = (x + m)(x + n)$.

iii. Verify by using FOIL expansion.

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Factoring (continued)

Factoring Trinomials, Leading Term ≠ x^2

To factor $ax^2 + bx + c,\ a \neq 1$:

By Grouping

i. Find m and n such that $mn = ac$ and $m + n = b$.

ii. Then $ax^2 + bx + c = ax^2 + mx + nx + c$.

iii. Group the first two terms and the last two terms.

iv. Follow the steps for factoring by grouping.

By Trial and Error

i. Factor a as pq and c as mn.

ii. For each such factorization, form the product $(px + m)(qx + n)$ and expand using FOIL.

iii. Stop when the expansion matches the original trinomial.

Remainder Theorem

If the polynomial $P(x)$ is divided by $x - a$, then the remainder is equal to $P(a)$.

Factor Theorem

For a polynomial $P(x)$ and number a, if $P(a) = 0$, then $x - a$ is a factor of $P(x)$.

SPECIAL FACTORIZATIONS

Difference of Squares

$$x^2 - y^2 = (x + y)(x - y)$$

Perfect Square Trinomials

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

Difference of Cubes

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Sum of Cubes

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

SOLVING QUADRATIC EQUATIONS BY FACTORING

Zero-Factor Property

If $ab = 0$, then $a = 0$ or $b = 0$.

Solving Quadratic Equations

i. Write in standard form:

$$ax^2 + bx + c = 0$$

ii. Factor.

iii. Use the zero-factor property to set each factor to zero.

iv. Solve each resulting equation to find each solution.

Rational Expressions

To find the value(s) for which a rational expression is undefined, set the denominator equal to 0 and solve the resulting equation.

Lowest Terms

To write a rational expression in lowest terms:

i. Factor the numerator and denominator.
ii. Divide out common factors.

OPERATIONS ON RATIONAL EXPRESSIONS

Multiplying Rational Expressions

i. Multiply numerators and multiply denominators.
ii. Factor numerators and denominators.
iii. Write expression in lowest terms.

Dividing Rational Expressions

i. Multiply the first rational expression by the reciprocal of the second rational expression.
ii. Multiply numerators and multiply denominators.
iii. Factor numerators and denominators.
iv. Write expression in lowest terms.

Finding the Least Common Denominator (LCD)

i. Factor each denominator into prime factors.
ii. List each different factor the greatest number of times it appears in any one denominator.
iii. Multiply the factors from step ii.

Writing a Rational Expression with a Specified Denominator

i. Factor both denominators.
ii. Determine what factors the given denominator must be multiplied by to equal the one given.
iii. Multiply the rational expression by that factor divided by itself.

Adding or Subtracting Rational Expressions

i. Find the LCD.
ii. Rewrite each rational expression with the LCD as denominator.
iii. If adding, add the numerators to get the numerator of the sum. If subtracting, subtract the second numerator from the first numerator to get the difference. The LCD is the denominator of the sum.
iv. Write expression in lowest terms.

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Rational Expressions (continued)

SIMPLIFYING COMPLEX FRACTIONS

Method 1

i. Simplify the numerator and denominator separately.
ii. Divide by multiplying the simplified numerator by the reciprocal of the simplified denominator.

Method 2

i. Multiply the numerator and denominator of the complex fraction by the LCD of all the denominators in the complex fraction.
ii. Write in lowest terms.

SOLVING EQUATIONS WITH RATIONAL EXPRESSIONS

i. Find the LCD of all denominators in the equation.
ii. Multiply each side of the equation by the LCD.
iii. Solve the resulting equation.
iv. Check that the resulting solutions satisfy the original equation.

Equations of Lines Two Variables

An ordered pair is a solution of an equation if it satisfies the equation.

If the value of either variable in an equation is given, the value of the other variable can be found by substitution.

GRAPHING LINEAR EQUATIONS

To graph a linear equation:

i. Find at least two ordered pairs that satisfy the equation.
ii. Plot the corresponding points. (An ordered pair (a, b) is plotted by starting at the origin, moving a units along the x-axis and then b units along the y-axis.)
iii. Draw a straight line through the points.

Special Graphs

$x = a$ is a vertical line through the point $(a, 0)$.

$y = b$ is a horizontal line through the point (a, b).

The graph of $Ax + By = 0$ goes through the origin. Find and plot another point that satisfies the equation, and then draw the line through the two points.

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Equations of Lines Two Variables (continued)

Intercepts

To find the x-intercept, let $y = 0$.

To find the y-intercept, let $x = 0$.

Slope

Suppose (x_1, y_1) and (x_2, y_2) are two different points on a line. If $x_1 \neq x_2$, then the slope is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

The slope of a vertical line is undefined.

The slope of a horizontal line is 0.

Parallel lines have the same slope.

Perpendicular lines have slopes that are negative reciprocals of each other.

EQUATIONS OF LINES

Slope–intercept form: $y = mx + b$,
where m is the slope, and $(0, b)$ is the y-intercept.

Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$,
where $(a, 0)$ is the x-intercept, and $(0, b)$ is the y-intercept.

Point–slope form: $y - y_1 = m(x - x_1)$,
where m is the slope and (x_1, y_1) is any point on the line.

Standard form: $Ax + By = C$

Vertical line: $x = a$

Horizontal line: $y = b$

Systems of Linear Equations

TWO VARIABLES

An ordered pair is a solution of a system if it satisfies all the equations at the same time.

Graphing Method

i. Graph each equation of the system on the same axes.
ii. Find the coordinates of the point of intersection.
iii. Verify that the point satisfies all the equations.

Substitution Method

i. Solve one equation for either variable.
ii. Substitute that variable into the other equation.
iii. Solve the equation from step ii.
iv. Substitute the result from step iii into the equation from step i to find the remaining value.

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Systems of Linear Equations (continued)

Elimination Method

i. Write the equations in standard form:
$$Ax + By = C.$$
ii. Multiply one or both equations by appropriate numbers so that the sum of the coefficient of one variable is 0.
iii. Add the equations to eliminate one of the variables.
iv. Solve the equation that results from step iii.
v. Substitute the solution from step iv into either of the original equations to find the value of the remaining variable.

Notes: If the result of step iii is a false statement, the graphs are parallel lines and there is no solution.

If the result of step iii is a true statement, such as $0 = 0$, the graphs are the same line, and the solution is every ordered pair on either line (of which there are infinitely many).

THREE VARIABLES

i. Use the elimination method to eliminate any variable from any two of the original equations.
ii. Eliminate the *same* variable from any *other* two equations.
iii. Steps i and ii produce a system of two equations in two variables. Use the elimination method for two-variable systems to solve for the two variables.
iv. Substitute the values from step iii into any of the original equations to find the value of the remaining variable.

APPLICATIONS

i. Assign variables to the unknown quantities in the problem.
ii. Write a system of equations that relates the unknowns.
iii. Solve the system.

MATRIX ROW OPERATIONS

i. Any two rows of the matrix may be interchanged.
ii. All the elements in any row may be multiplied by any nonzero real number.
iii. Any row may be modified by adding to the elements of the row the product of a real number and the elements of another row.

A system of equations can be represented by a matrix and solved by matrix methods. Write an augmented matrix and use row operations to reduce the matrix to row echelon form.

Inequalities and Absolute Value: One Variable

Properties

i. **Addition:** The same quantity may be added to (or subtracted from) each side of an inequality without changing the solution.
ii. **Multiplication by positive numbers:** Each side of an inequality may be multiplied (or divided) by the same positive number without changing the solution.
iii. **Multiplication by negative numbers:** If each side of an inequality is multiplied (or divided) by the same negative number, the direction of the inequality symbol is reversed.

Solving Linear Inequalities

i. Simplify each side separately.
ii. Isolate the variable term on one side.
iii. Isolate the variable. (Reverse the inequality symbol when multiplying or dividing by a negative number.)

Solving Compound Inequalities

i. Solve each inequality in the compound, inequality individually.
ii. If the inequalities are joined with *and*, then the solution set is the **intersection** of the two individual solution sets.
iii. If the inequalities are joined with *or*, then the solution set is the **union** of the two individual solution sets.

Solving Absolute Value Equations and Inequalities

Suppose k is positive.

To solve $|ax + b| = k$, solve the compound equation
$$ax + b = k \quad \text{or} \quad ax + b = -k.$$

To solve $|ax + b| > k$, solve the compound inequality
$$ax + b > k \quad \text{or} \quad ax + b < -k.$$

To solve $|ax + b| < k$, solve the compound inequality
$$-k < ax + b < k.$$

To solve an absolute value equation of the form $|ax + b| = |cx + d|$, solve the compound equation
$$ax + b = cx + d \quad \text{or}$$
$$ax + b = -(cx + d).$$

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Inequalities and Absolute Value: One Variable (continued)

Graphing a Linear Inequality

i. If the inequality sign is replaced by an equals sign, the resulting line is the equation of the boundary.
ii. Draw the graph of the boundary line, making the line *solid* if the inequality involves $\leq$ or $\geq$ or *dashed* if the inequality involves $<$ or $>$.
iii. Choose any point not on the line as a test point and substitute its coordinates into the inequality.
iv. If the test point satisfies the inequality, shade the region that includes the test point; otherwise, shade the region that does not include the test point.

Functions

Function Notation

A **function** is a set of ordered pairs (x, y) such that for each first component x, there is one and only one second component y. The set of first components is called the **domain**, and the set of second components is called the **range**.

$y = f(x)$ defines y as a function of x.

To write an equation that defines y as a function of x in function notation, solve the equation for y and replace y by $f(x)$.

To evaluate a function written in function notation for a given value of x, substitute the value wherever x appears.

Variation

If there exists some real number (constant) k such that:

$y = kx^n$, then y varies directly as x^n.

$y = \dfrac{k}{x^n}$, then y varies inversely as x^n.

$y = kxz$, then y varies jointly as x and z.

Operations on Functions

If $f(x)$ and $g(x)$ are functions, then the following functions are derived from f and g:
$$(f + g)(x) = f(x) + g(x)$$
$$(f - g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x) \cdot g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \; g(x) \neq 0$$

Composition of f and g:
$$(f \circ g)(x) = f[g(x)]$$

Roots and Radicals

Radical Expressions and Graphs

$\sqrt[n]{a} = b$ means $b^n = a$.

$\sqrt[n]{a}$ is the principal or positive nth root of a.

$-\sqrt[n]{a}$ is the negative nth root of a.

$\sqrt[n]{a^n} = |a|$ if n is even.

$\sqrt[n]{a^n} = a$ if n is odd.

Rational Exponents

$a^{1/n}$: If $\sqrt[n]{a}$ is real, then $a^{1/n} = \sqrt[n]{a}$.

$a^{m/n}$: If m and n are positive integers with m/n in lowest terms and $a^{1/n}$ is real, then

$$a^{m/n} = (a^{1/n})^m.$$

If $a^{1/n}$ is *not* real, then $a^{m/n}$ is *not* real.

SIMPLIFYING RADICAL EXPRESSIONS

Product Rule: If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real and n is a natural number, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.$$

Quotient Rule: If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real and n is a natural number, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

OPERATIONS ON RADICAL EXPRESSIONS

Adding and Subtracting: Only radical expressions with the *same index* and the *same radicand* can be combined.

Multiplying: Multiply binomial radical expressions by using FOIL expansion.

Dividing: Rationalize the denominator by multiplying both the numerator and denominator by the same expression. If the denominator involves the sum of an integer and a square root, the expression used will be chosen to create a difference of squares.

Solving Equations Involving Radicals

i. Isolate one radical on one side of the equation.

ii. Raise both sides of the equation to a power that equals the index of the radical.

iii. Solve the resulting equation; if it still contains a radical, repeat steps i and ii.

iv. The resulting solutions are only candidates. Check which ones satisfy the original equation. Candidates that do not check are extraneous (not part of the solution set).

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Roots and Radicals (continued)

COMPLEX NUMBERS

The imaginary unit is $i = \sqrt{-1}$, so $i^2 = -1$.

For $b > 0$, $\sqrt{-b} = i\sqrt{b}$. To multiply radicals with negative radicands, first change each factor to the form $i\sqrt{b}$.

A complex number has the form $a + bi$, where a and b are real numbers.

OPERATIONS ON COMPLEX NUMBERS

Adding and Subtracting Complex Numbers

Add (or subtract) the real parts and add (or subtract) the imaginary parts.

Multiplying Complex Numbers

Multiply using FOIL expansion and using $i^2 = -1$ to reduce the result.

Dividing Complex Numbers

Multiply the numerator and the denominator by the conjugate of the denominator.

Quadratic Equations, Inequalities, and Functions

SOLVING QUADRATIC EQUATIONS

Square Root Property

If a is a complex number, then the solutions to $x^2 = a$ are $x = \sqrt{a}$ and $x = -\sqrt{a}$.

Solving Quadratic Equations by Completing the Square

To solve $ax^2 + bx + c = 0$, $a \neq 0$:

i. If $a \neq 1$, divide each side by a.

ii. Write the equation with the variable terms on one side of the equals sign and the constant on the other.

iii. Take half the coefficient of x and square it. Add the square to each side of the equation.

iv. Factor the perfect square trinomial and write it as the square of a binomial. Combine the constants on the other side.

v. Use the square root property to determine the solutions.

Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$b^2 - 4ac$ is called the **discriminant** and determines the number and type of solutions.

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Quadratic Equations, Inequalities, and Functions (continued)

Discriminant	Number and Type of Solution
$b^2 - 4ac > 0$	Two real solutions
$b^2 - 4ac = 0$	One real solution
$b^2 - 4ac < 0$	Two complex solutions

QUADRATIC FUNCTIONS

Standard Form

$f(x) = ax^2 + bx + c$, for a, b, c real, $a \neq 0$.

The graph is a parabola, opening *up* if $a > 0$, *down* if $a < 0$. The vertex is $\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$.

The axis of symmetry is $x = \frac{-b}{2a}$.

Vertex Form

$f(x) = a(x - h)^2 + k$. The vertex is (h, k). The axis of symmetry is $x = h$.

Horizontal Parabola

The graph of $x = ay^2 + by + c$, is a horizontal parabola, opening to the *right* if $a > 0$, to the *left* if $a < 0$. Note that this is *not* the graph of a function.

QUADRATIC INEQUALITIES

Solving Quadratic (or Higher-Degree Polynomial) Inequalities

i. Replace the inequality sign by an equality sign and find the real-valued solutions to the equation.

ii. Use the solutions from step i to divide the real number line into intervals.

iii. Substitute a test number from each interval into the original inequality to determine the intervals that belong to the solution set.

iv. Consider the endpoints separately.

Inverse, Exponential, and Logarithmic Functions

Inverse Functions

If any horizontal line intersects the graph of a function in, at most, one point, then the function is one to one and has an inverse.

If $y = f(x)$ is one to one, then the equation that defines the inverse function f^{-1} is found by interchanging x and y, solving for y, and replacing y with $f^{-1}(x)$.

The graph of f^{-1} is the mirror image of the graph of f with respect to the line $y = x$.

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Inverse, Exponential, and Logarithmic Functions (continued)

Exponential Functions

For $a > 0$, $a \neq 1$, $f(x) = a^x$ defines the exponential function with base a.

Properties of the graph of $f(x) = a^x$:

i. Contains the point (0, 1)

ii. If $a > 1$, the graph *rises* from left to right. If $0 < a < 1$, the graph *falls* from left to right.

iii. The x-axis is an asymptote.

iv. Domain: $(-\infty, \infty)$; Range: $(0, \infty)$

Logarithmic Functions

The logarithmic function is the inverse of the exponential function:

$$y = \log_a x \quad \text{means} \quad x = a^y.$$

For $a > 0$, $a \neq 1$, $g(x) = \log_a x$ defines the logarithmic function with base a.

Properties of the graph of $g(x) = \log_a x$:

i. Contains the points (1, 0) and (a, 1)

ii. If $a > 1$, the graph *rises* from left to right. If $0 < a < 1$, the graph *falls* from left to right.

iii. The y-axis is an asymptote.

iv. Domain: $(0, \infty)$; Range: $(-\infty, \infty)$

Logarithm Rules

Product rule: $\log_a xy = \log_a x + \log_a y$

Quotient rule: $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power rule: $\log_a x^r = r \log_a x$

Special properties: $a^{\log_a x} = x$, $\log_a a^x = x$

Change-of-base rule: For $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$, $x > 0$, $\log_a x = \dfrac{\log_b x}{\log_b a}$.

Exponential, Logarithmic Equations

Suppose $b > 0$, $b \neq 1$.

i. If $b^x = b^y$, then $x = y$.

ii. If $x > 0$, $y > 0$, then $\log_b x = \log_b y$ is equivalent to $x = y$.

iii. If $\log_b x = y$, then $b^y = x$.

Conic Sections and Nonlinear Systems

CIRCLE

Equation of a Circle: Center-Radius

$$(x - h)^2 + (y - k)^2 = r^2$$

is the equation of a circle with radius r and center at (h, k).

Equation of a Circle: General

$$x^2 + y^2 + ax + by + c = 0$$

Given an equation of a circle in general form, complete the squares on the x and y terms separately to put the equation into center-radius form.

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Conic Sections and Nonlinear Systems (continued)

ELLIPSE

Equation of an Ellipse (Standard Position, Major Axis along *x*-axis)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b > 0$$

is the equation of an ellipse centered at the origin, whose x-intercepts (vertices) are $(a, 0)$ and $(-a, 0)$ and y-intercepts are $(0, b)$ $(0, -b)$. Foci are $(c, 0)$ and $(-c, 0)$, where $c = \sqrt{a^2 - b^2}$.

Equation of an Ellipse (Standard Position, Major Axis along *y*-axis)

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1, \; a > b > 0$$

is the equation of an ellipse centered at the origin, whose x-intercepts (vertices) are $(b, 0)$ and $(-b, 0)$ and y-intercepts are $(0, a)$ $(0, -a)$. Foci are $(0, c)$ and $(0, -c)$, where $c = \sqrt{a^2 - b^2}$.

HYPERBOLA

Equation of a Hyperbola (Standard Position, Opening Left and Right)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

is the equation of a hyperbola centered at the origin, whose x-intercepts (vertices) are $(a, 0)$ and $(-a, 0)$. Foci are $(c, 0)$ and $(-c, 0)$, where $c = \sqrt{a^2 + b^2}$. Asymptotes are $y = \pm \frac{b}{a} x$.

Equation of a Hyperbola (Standard Position, Opening Up and Down)

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

is the equation of a hyperbola centered at the origin, whose y-intercepts (vertices) are $(0, a)$ and $(0, -a)$. Foci are $(0, c)$ and $(0, -c)$, where $c = \sqrt{a^2 + b^2}$. Asymptotes are $y = \pm \frac{a}{b} x$.

SOLVING NONLINEAR SYSTEMS

A nonlinear system contains multivariable terms whose degrees are greater than one.

A nonlinear system can be solved by the substitution method, the elimination method, or a combination of the two.

Sequences and Series

A sequence is a list of terms $t_1, t_2, t_3, \ldots$ (finite or infinite) whose general (nth) term is denoted t_n.

A series is the sum of the terms in a sequence.

ARITHMETIC SEQUENCES

An arithmetic sequence is a sequence in which the difference between successive terms is a constant.

Let a_1 be the first term, a_n be the nth term, and d be the common difference.

Common difference: $d = a_{n+1} - a_n$

***n*th term:** $a_n = a_1 + (n - 1)d$

Sum of the first *n* terms:

$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n - 1)d]$$

GEOMETRIC SEQUENCES

A geometric sequence is a sequence in which the ratio of successive terms is a constant.

Let t_1 be the first term, t_n be the nth term, and r be the common ratio.

Common ratio: $r = \dfrac{t_{n+1}}{t_n}$

***n*th term:** $t_n = t_1 r^{n-1}$

Sum of the first *n* terms:

$$S_n = \frac{t_1(r^n - 1)}{r - 1}, \; r \neq 1$$

Sum of the terms of an infinite geometric sequence with $|r| < 1$: $S = \dfrac{t_1}{1 - r}$

The Binomial Theorem

Factorials

For any positive integer n,
$n! = n(n - 1)(n - 2) \ldots (3)(2)(1)$
and
$0! = 1$.

Binomial Coefficient

For any nonnegative integers n and r, with $r \leq n$, $\binom{n}{r} = {}_nC_p = \dfrac{n!}{r!(n - r)!}$.

The binomial expansion of $(x + y)^n$ has $n + 1$ terms. The $(r + 1)$st term of the binomial expansion of $(x + y)^n$ for $r = 0, 1, \ldots, n$ is

$$\frac{n!}{r!(n - r)!} x^{n-r} y^r.$$